FERMILAB-Pub-85/53-A
April 1985

INFLATION AND NON-MINIMAL SUPERGRAVITY

Lars Gerhard Jensen

and

Keith A. Olive

Theoretical Astrophysics Group
Fermi National Accelerator Laboratory
P.O. Box 500
Batavia, IL 60510 U.S.A.

ABSTRACT

Inflation based on a single chiral superfield (the inflaton) in N=1 supergravity has been shown to be incompatible with the so-called thermal constraint requiring a high temperature minimum at a point at which the zero temperature scalar potential is flat, in a theory using fields with minimal scalar kinetic terms. Here we show that by modifying the kinetic terms of the inflaton, one can satisfy the thermal constraint without introducing additional fields or small parameters.

Submitted to Nuclear Physics B 4/85.

I. INTRODUCTION

Inflationary cosmologies [1] have been shown to be capable of resolving a number of cosmological problems such as the horizon problem and the flatness/oldness problem. Inherent in these models is that the universe enters a de Sitter-like phase [1,2] during which the scale factor of the universe (R) increases exponentially. The de Sitter phase can be realized in models where the universe undergoes a first order phase transition during which it supercools many orders of magnitude. One of the basic assumptions in the standard hot big bang cosmology, that the universe expands adiabatically, can be strongly violated by such a transition, since following the extreme supercooling the latent heat of the transition is released. This entropy production can potentially lead to a solution of the cosmological horizon, flatness/oldness problems associated with the standard model [1].

The new inflationary universe [3] was the first serious candidate for a realistic inflationary cosmology scenario. Here the inflation (exponential expansion) is triggered by the spontaneous breakdown of a gauge symmetry in a grand unified theory (GUT), and it was shown that GUT models with the Higgs potential of the Coleman-Weinberg type can produce the amount of inflation required to solve the cosmological problems when mass scales are correctly tuned. The new inflationary universe however fails on an important point: the amplitude of the spectrum of energy density fluctuations $\delta\rho/\rho$ comes out much too large to be consistent with galaxy formation [4]. Other problems have also been shown to plague this scenario [5,6]. This disappointment has led many to the study of inflation in the framework of supersymmetry [6-8]. In supersymmetric theories, radiative correction to the Higgs potential are small and a natural explanation of the hierarchy problem along with small stable masses needed for inflation may be possible. In this paper we shall be concerned about a special class of supersymmetric inflationary theories; those in which the scale of the "symmetry breaking" is of the order of $M=M_p/\sqrt{8\pi}=2.4\times10^{18}$ GeV, where M_p is the Planck scale (this is called primordial inflation) [7]. In the models we consider, inflation is not associated with the breakdown of any symmetry as in the new inflationary universe. However, much in analogy with the latter, inflation is generated in a first order phase transition during which a scalar field (the inflaton) acquires a non zero vacuum expectation value $\langle\phi\rangle=v\approx M$. The scalar potential V is fine tuned so that the cosmological constant vanishes at v, i.e. $V(v)=0$. It has been argued that primordial inflation (i.e. inflation above the GUT scale) may be more natural than inflation at lower scales such as at the GUT scale, because it involves less fine tuning of parameters. The reason being that flat potentials, as are needed in the inflationary picture, are easier to obtain as $v\to M$.

The combination of primordial inflation and supersymmetry naturally leads one to consider inflation in supergravity. Several authors have already studied inflation in N=1 minimal supergravity models, but it appears difficult if not impossible to satisfy all the constraints an inflationary model must satisfy [9]. By minimal supergravity models we mean models in which $G_i^j=\delta_i^j$ (here G is the Kähler potential), so that the kinetic terms of the scalars, $G_i^j\partial_\mu\phi^i\partial^\mu\phi_j^*/2$ are the usual ones. It is our hope that by considering more general Kähler potentials, with G_i^j an arbitrary positive definite tensor (non minimal supergravity), the inflationary constraints can be satisfied. In sections to follow, we shall argue that this is actually possible, and we shall provide an example of an inflationary cosmology which satisfies the constraints without any fine tuning (except for the cosmological constant). We shall also provide an example in the large N limit (N being the number of scalar fields in addition to the inflaton), in which all constraints are satisfied.

In Section II we review some results from minimal supergravity models, and consider in detail the constraint imposed through finite temperature corrections to the scalar potential. In Section III we define non-minimal supergravity in more detail and present our results. Our conclusions are given in Section IV.

II. MINIMAL SUPERGRAVITY

NOTATION: We begin this section by introducing the notation used throughout this paper. We shall consider scalar fields ϕ^i, i=0, 1,...., N, and potentials $u(\phi^i,\phi^*_j)$. Here ϕ^o is the inflaton, and we simply write ϕ for ϕ^o. For derivatives we use the following abbreviations, $u^{lm..}_{ij..}$ = $(\partial/\partial\phi^i)(\partial/\partial\phi^j)..(\partial/\partial\phi^*_m)\ (\partial/\partial\phi^*_l)u$, and sometimes also $u_\phi = u_o$, $u^\phi = u^o$ etc. when we are concerned about derivatives with respect to ϕ and ϕ^*.

Given the Lagrangian for N=1 supergravity coupled to matter [10], one can write down the most general form for the scalar potential in terms of the Kähler potential G,

$$V = \exp\ (G)[G_i(G^{-1})^i_jG^j - 3] \qquad (2.1)$$

where G is a real function of the chiral superfields ϕ^i. Our units through this paper will be $M = M_p/\sqrt{8\pi} = 1$. The scalar kinetic terms of the theory are given in terms of G as well

$$\mathcal{L}_{K.T} = -G_i^j(\partial_\mu\phi^i)(\partial^\mu\phi_j^*)/2 \tag{2.2}$$

Hence, we will use the term minimal N=1 supergravity to describe those theories which have a priori normalized kinetic terms or

$$G_i^j = \delta_i^j \tag{2.3}$$

It is common in minimal theories, to express the Kähler potential in terms of another arbitrary function called the superpotential $F(\phi^i)$,

$$G = \phi^i\phi_i^* + \log|F|^2 \tag{2.3}$$

In this case the scalar potential takes its more familiar form

$$V = \exp(\phi^i\phi_i^*)[|F_i + \phi_i^*F|^2 - 3|F|^2] \tag{2.4}$$

The first attempt [11] to write down a model for inflation in the context of supergravity, considered a single chiral superfield ϕ to be known as the inflaton. Starting with an arbitrary polynomial for $F(\phi)$

$$F(\phi) = \mu^2 \sum_{n=0}^{\infty} \lambda_n\phi^n \tag{2.5}$$

one hoped to use the constraints on the scalar potential for inflation to set conditions on the couplings λ_i. The two major constraints (a complete list of constraints will be given in Section III) require a long rollover timescale, that is the timescale for the inflaton to pick up its vacuum expectation value v = 1 and an acceptable magnitude for the density perturbation $\delta\rho/\rho$ produced during the rollover. This last constraint will fix the magnitude of the mass scale μ in the superpotential (2.5).

One notices that the higher order terms in F apparently introduce non-renormalizable interactions, but because we will view this as an effective theory, these terms are suppressed below the Planck scale by powers of M, and above the Planck scale new physics is hoped to cure this problem. The superpotential F is in general a sum of several pieces,

$$F = F_I + F_G + F_S \tag{2.6}$$

due to the inflaton, the GUT sector and the SUSY breaking sector. Since we are concerned about primordial inflation, we assume that at large energy scales the inflaton piece dominates. It is only at lower energy scales, after inflation, that the GUT piece and then the SUSY piece contributes to F. In what follows we will concentrate only on the effects due to F_I.

A simple example of an inflationary model in minimal supergravity at zero temperature which satisfies all constraints is [12]

$$F(\phi) = \mu^2(1-\phi)^2 \tag{2.7}$$

(i.e., $\lambda_o=1$, $\lambda_1=-2$, and $\lambda_2=1$). This choice of F insures that the first and second derivatives of V vanish at $\phi=0$ as is required for sufficient inflation, that V(0) is positive and that V has a minimum with vanishing cosmological constant at $\phi=1$. With a suitable choice of μ it is also possible to satisfy the constraint imposed by the energy density fluctuations, that $\delta\rho/\rho \approx 10^{-4}$. Thus the zero temperature potential V derived from this F satisfies all requirements for a successful inflationary cosmology.

We note here that the minimum of the potential at $\phi=1$ preserves supersymmetry. The condition for supersymmetry breaking being

$$e^{G/2}G_i(G^{-1})^i_j \neq 0 \tag{2.8}$$

for some field j. If for example, the global minimum for the inflaton did break supersymmetry, the supersymmetry breaking scale M_S would be determined by the single scale in F namely

$$M_S \sim \mu^2 \ . \tag{2.9}$$

As we will see, μ is determined by $\delta\rho/\rho$ to be $\mu \sim 10^{-4}$ so that $M_S \sim 10^{-8} \sim 10^{10}$ GeV. This scale is much too large for supersymmetry to be useful as a solution to the gauge hierarchy problem. Adjusting $\mu \sim 10^{-8}$ to give correct supersymmetry breaking then leads to an unacceptable inflationary model [13]. Thus in order to avoid extreme fine tuning and the introduction of several scales in the superpotential, we will only consider models in which the inflaton preserves supersymmetry at the global minimum and models in which F_I is a function of a single field and carries only a single scale μ and all couplings λ_i are $O(1)$.

The model given by eq. (2.7) is indeed quite simple. However a problem arises when one considers the initial conditions for the inflaton ϕ. Without specifying the initial conditions by hand, the only way to determine them is by examining corrections to $V(\phi)$ at high temperatures [14]. However if one calculates the finite temperature potential V_T using the same superpotential (2.7) one finds that V and V_T do not have extrema at the same values of ϕ, in particular at $\phi=0$. This means that the value of ϕ as the temperature cools below the critical temperature may be different from zero, and only if V has vanishing first and second derivatives at this value of ϕ, may one hope to get inflation. We therefore conclude that in order to have a consistent scenario, we must impose the thermal constraint i.e., the constraint that V_T has a minimum at $\phi=0$ (or to be more precise, at the value of ϕ at which the first and second derivatives of V vanish, i.e. where $V(\phi)$ is flat).

It has been argued [15] that in the case we are considering, single field inflation in minimal supergravity, it is not possible to satisfy the inflationary and thermal constraints simultaneously. Indeed, one can show that the requirements $V(0) > 0$, $\partial V/\partial\phi\ (0) = 0$ and $\partial V_T/\partial\phi\ (0) = 0$ are incompatible. The expression for V_T has been calculated,

$$V_T = -\pi^2 N_B T^4/48 + \mathrm{Tr}(m_B^2+m_F^2/2)T^2/24 \tag{2.10}$$

where N_B is the number of boson degrees of freedom and m_B^2 (m_F^2) is the boson (fermion) mass-squared matrix. These matrices depend on the Kähler potential G and its derivatives, which again depend on the vacuum expectation values of the scalar fields. In terms of the Kähler potential we have the following expressions for the traces involved in the calculation of V_T [16]

$$\begin{aligned} \mathrm{Tr}(m_B^2) &= 2(G^{-1})^i_j V^j_i \\ \mathrm{Tr}(m_F^2) &= 2e^G[|(G^{-1})^k_i\ (G_{kj} + G_k G_j - G_\ell G^p_{kj}(G^{-1})^\ell_p)|^2 - 2] \end{aligned} \tag{2.11}$$

where we have included the spin-1/2 and the spin-3/2 (gravitino) contributions in the fermion trace. At high temperatures the dominating field-dependent term in the effective potential, $V+V_T$, is V_T, and its minimum determines the vacuum expectation values of the ϕ^i's at high temperatures.

In the case we are considering ($G^j_i=\delta^j_i$) eq. (2.10) can be simplified,

$$V_T = T^2 e^G[3/2\ (A+B+C^2) + (N-1)C - (2N+1)]/12 \tag{2.12}$$

where

$$A = G_i G^{ij} G_j + \mathrm{h.c.} \tag{2.13a}$$

$$B = G_{ij}G^{ij} \tag{2.13b}$$

$$C = G_iG^i \tag{2.13c}$$

and N is the total number of chiral supermultiplets. In the limit that N is large, eq. (2.12) further simplifies to [16]

$$V_T = NT^2e^G(C-2)/12 \tag{2.14}$$

We remind the reader that in this notation we have

$$V = e^G(C-3) \tag{2.15}$$

for the case under consideration.

If we now try to apply the condition $\partial V/\partial\phi\ (0) = \partial V_T/\partial\phi\ (0) = 0$, we have

$$G_\phi e^G(C-2) = G_\phi e^G(C-3) \tag{2.16}$$

which can be satisfied only if $G_\phi = 0$ or $e^G = 0$. However these solutions correspond to $V(0)<0$ and $V(0)=0$ respectively and both violate the necessary condition that $V(0)>0$. Hence the incompatibility between the inflationary constraints and the thermal constraint.

The observant reader may object to the fact that we have required a thermal minimum at $\phi=0$ rather than $\phi\approx0$ which would be sufficient for our purposes. This case however turns out to invoke unnatural means such as fine tuning coupling constants of the scalar field interactions by many orders of magnitude. Indeed even efforts to satisfy all constraints by including extra fields interacting with the inflaton [17] have led to severe fine tuning in the superpotential. The only way we hope to remedy this situation is to examine non-minimal models.

III. NON-MINIMAL SUPERGRAVITY

In this paper we shall try to examine the possibility of satisfying the thermal constraint by considering non-minimal supergravity. Non-minimal supergravity has been considered before in other contexts by several authors [18]. As we have said earlier, what we mean by non-minimal supergravity is the class of theories in which $G_i^j \neq \delta_i^j$. Because the kinetic terms of the scalar fields (given by eq. (2.2)) are no longer correctly normalized, a transformation of the fields ϕ^i to new (physical) fields are required. This brings the kinetic terms back to normal form in terms of the new fields.

We will assume that only one chiral superfield whose scalar component ϕ is called the inflaton, is responsible for inflation and that the effect of all other fields are negligible at the energy scale at which the inflation occurs. (To be more precise, the inflaton is the real part, ϕ_R, of the complex field $\phi=\phi_R+i\phi_I$.) We first consider the case where the inflaton is the only field present (note that we do not at any time impose a priori that $\phi=\phi_R$, instead we will impose conditions for the stability of $\langle\phi_I\rangle=0$). Later we consider the so called large N limit.

The equation of motion satisfied by the inflaton ϕ may be derived from the Lagrangian

$$L = \int d^4x\sqrt{-g}(G_i^j\partial_\mu\phi^i\partial_\nu\phi_j^*g^{\mu\nu}/2 - V(\phi,\phi^*)) \quad (3.1)$$

We find that ϕ - assumed to be homogeneous - satisfies the scalar field equation

$$G_\phi^\phi(\ddot\phi+3H\dot\phi) + G_{\phi\phi}^\phi\dot\phi^2 = -V^\phi \quad , \quad (3.2)$$

where we have also assumed that the space-time metric $g_{\mu\nu}$ has the usual Robertson-Walker form, and H is the Hubble constant. When we examine this equation for the purpose of inflation it is important to notice that in theories with non-minimal kinetic terms the effective "force" which drives ϕ to the minimum of V is $-V^\phi/G_\phi^\phi$ instead of the usual $-V^\phi$. Notice also that an extra "friction-term" appears in the equation of motion.

We shall attempt to devise a $G(\phi,\phi^*)$ so as to obtain an effective potential satisfying all constraints including the thermal one. We will choose G in such a manner that G_ϕ^ϕ equals 1 to at least second order at $\phi=0$, so that we can replace G_ϕ^ϕ with 1 when we impose the constraints at $\phi=0$ required to obtain successful inflation. The zero temperature potential V should have a shape like shown in figure 1, where V is positive and monotoneously decreasing on the real ϕ-axis between $\phi_R=0$ and $\phi_R=v$. V must be chosen to be very flat at $\phi=0$ in order to provide for sufficient inflation, so we demand that

$$V > 0 \tag{3.3}$$

$$\frac{\partial V}{\partial \phi_R} = 0 \tag{3.4}$$

$$\frac{\partial^2 V}{\partial \phi_R^2} = 0 \tag{3.5}$$

at $\phi_R = \phi_I = 0$. The imaginary part, ϕ_I of ϕ will be irrelevant for inflation, so in order to simplify matters, we require that ϕ_I is stable during inflation, i.e. that

$$\frac{\partial V}{\partial \phi_I} = 0 \tag{3.6}$$

$$\frac{\partial}{\partial \phi_I}\left((G^{\phi}_{\phi})^{-1}\frac{\partial V}{\partial \phi_I}\right) > 0 \tag{3.7}$$

for all $\phi=\phi^*$ (notice that this is equivalent to $\phi_I = 0$ and ϕ_R arbitrary).

Furthermore we must choose G so that the cosmological constant vanishes at the minimum, i.e. we require that

$$V(\phi_R = v,\ \phi_I = 0) = 0 \tag{3.8}$$

In addition as we have said in the previous section, we wish to impose, that ϕ has nothing to do with the breaking of supersymmetry. Therefore we demand that the minimum of V at v be supersymmetry conserving; this amounts to the requirement eq. (2.8) that

$$e^{G/2}G_{\phi} = 0 \tag{3.9}$$

at $\phi=\phi^{*}=v$.

In order to satisfy the thermal constraint we must require that V_T has a stable minimum at the point where V is flat (in this case at $\phi=0$), i.e., that the first derivatives of V_T vanish and that the mass matrix m_T (calculated from V_T) is positive definite at this point,

$$\frac{\partial V_T}{\partial \phi_R} = \frac{\partial V}{\partial \phi_I} = 0 \tag{3.10}$$

$$m_T^2 = \frac{\partial^2 V}{\partial \phi_i \partial \phi_j} > 0 \tag{3.11}$$

for i,j = R or I at $\phi = 0$. Thus eqs. (3.3) - (3.11) represent the constraints we will require on $G(\phi,\phi^*)$ which can provide a scalar potential capable of inflation. Other constraints such as density perturbations and reheating will be discussed below.

In order to satisfy these constraints we write G in the following form

$$G(\phi,\phi^*) = g(\phi,\phi^*) + \log|F(\phi)|^2 \quad , \tag{3.12}$$

where

$$F(\phi) = \mu^2[1-(\tfrac{\phi}{v})^n]^m \tag{3.13}$$

and where g is a real function, $g(\phi,\phi^*) = \Sigma a_{kl}\phi^k\phi^{*l}$ with $a_{kl} = a^*_{lk}$. We have here restricted ourselves to the case where the a_{kl}'s are real; this insures that V too is a power series in ϕ and ϕ^* with real coefficients, and therefore that $\partial V/\partial\phi_I$ vanishes when $\phi=\phi^*$ (see eq. (3.5)). This particular form of F is chosen in order to trivially satisfy the constraints at the minimum $\phi = v$; this choice insures that the supersymmetry constraint (eq. 3.8) is satisfied, and since V contains a prefactor of $\exp(G)$, it is apparent that V vanishes at $\phi = v$.

We now make the following choice of g:

$$
\begin{aligned}
&g(\phi,\phi^*) = \\
&a(\phi+\phi^*)+\phi\phi^*+(2-a^2)(\phi^2+\phi^{*2})/2+ (ab-5a-a^2c/3+2a^3)(\phi^3+\phi^{*3})/6 \\
&\qquad +(1-ac/3)(\phi\phi^*)^2/4+b(\phi^3\phi^*+\phi\phi^{*3})/6+ c(\phi^3\phi^{*2}+\phi^2\phi^{*3})/12 \ .
\end{aligned}
\tag{3.14}
$$

We have for simplicity chosen $a_{12} = 0$ (i.e. the coefficient of $\phi\phi^*$ $(\phi+\phi^*)$). With this choice of g all the remaining constraints except for the ones involving inequalities are satisfied. Therefore what remains to be done is to choose - if possible - a, b, c, m, n and v so the inequalities are satisfied.

We have attempted this with the aid of a computer and found that it is in fact possible to satisfy all constraints for example by choosing

$$(a,b,c,m,n,v) = (-2,4,-3,2,5,1) \ . \tag{3.15}$$

The $T=0$ scalar potential for these parameters is of the same form as in Fig. 1.

We have not yet included the constraint coming from energy density perturbation on the potential. For a potential of the form

$$V(\phi) = \mu^4(\lambda - \lambda_2\phi^3/3 + \ldots) \tag{3.16}$$

(as is the case for $\phi \approx 0$ for G defined by (3.12-3.14), the expression for the energy density perturbations is [4,19]

$$\frac{\delta\rho}{\rho} \approx \frac{1}{(6\pi^3\lambda)^{1/2}}\, \mu^2\lambda_2 \, \ln^2(Hk^{-1}) \tag{3.17}$$

where k is the wave number of the perturbation, and $H = (\lambda/\sqrt{3})^{1/2}$ with λ and λ_2 of order unity, and galactic size perturbations eq. (3.18) becomes

$$\frac{\delta\rho}{\rho} \sim 10^3\mu^2 \tag{3.18}$$

implying that $\mu \sim 10^{-7} - 10^{-8}$ so that $\delta\rho/\rho \sim 10^{-4} - 10^{-5}$ in agreement with limits from the anisotropy of the microwave background radiation.

Finally, as we have remarked above, we must transform the fields into physical fields. In the appendix we have shown how this is done for a G of the form we have chosen above, and we argue that the constraints are still satisfied for the new (physical) fields.

We shall now consider the effects of additional fields and the large N limit. In a realistic model, the GUT sector provides for on the order of 100 extra fields, which contributes to the Kähler potential, so that in general V and V_T are changed. We assume for simplicity that G_i^j is diagonal and that $G_i^j = \delta_i^j$ for $i,j=1,2,\dots,N$. We also let $\phi=\phi^o$ be the inflaton. Then we find for the new potentials

$$V = \exp(G)[|G_\phi|^2/G_\phi^\phi-3] \tag{3.19}$$

and

$$V_T = V_T^o + NT^2(V+\exp(G))/12 \; ; \tag{3.20}$$

where we have only included the part which depends on the inflaton (ϕ,ϕ^*), V_T^o is the previously used expression for the thermal potential, and we notice that V is unchanged as a function of G, but that due to the extra fields, V_T contains additional terms of which we only have included the leading term in N. In order to satisfy the constraints we make the following ansatz for the ϕ-dependent part of G, $G = g(\phi,\phi^*)+\log|F(\phi)|^2$, and

$$g(\phi,\phi^*) =$$
$$a(\phi+\phi^*)+\phi\phi^*+(2-a^2)(\phi^2+\phi^{*2})/2 + (ab-5a-a^2c/3+2a^3)(\phi^3+\phi^{*3})/6 \tag{3.21}$$
$$+(1-ac/3)(\phi\phi^*)^2/4+b(\phi^3\phi^*+\phi\phi^{*3})/6 + (c+N/a)(\phi^3\phi^{*2}+\phi^2\phi^{*3})/12.$$

This choice satisfies all constraints except the ones involving inequalities. These must be satisfied by a suitable choice of the remaining parameters a, b, c, m, n and v. N is typically of the order

100; for illustrative purposes we have chosen N=100. Again, with the aid of a computer, we have found it possible to satisfy all constraints by a suitable choice of parameters. For example the following set is sufficient $(a,b,c,m,n,v) = (-3,15,-2,2,5,1/2)$. Analogously to the case with only one field, a field transformation is needed. This transformation only involves ϕ and ϕ^* since only G^ϕ_ϕ is non-trivial and $G^j_i = \delta^j_i$; see the appendix.

For completeness, we observe that reheating in these models does not present any additional problems. The inflaton is coupled to ordinary matter only through gravity and it decays with a decay rate $\Gamma \propto M^3_\phi/M^2$. In the examples above, M_ϕ is always of the order $\mu^2 M$ so that the reheat temperature is [20,9,12]

$$T_R \sim (\Gamma M)^{1/2} \sim \mu^3 M \sim 10^6 \text{ GeV} \tag{3.22}$$

Although this may seem like a low reheating temperature, if the mass of this Higgs boson responsible for generating the baryon asymmetry is $M_H > 10^{10}$ GeV, the net baryon number produced is

$$\frac{n_B}{s} \approx \frac{T_R}{M_H} (\Delta B) \sim 10^{-4}(\Delta B) \tag{3.23}$$

where ΔB is the baryon number produced by H, H^* decay. (See however Ref. 21.)

We conclude this section by discussing how to extend the above to include other sectors such as the GUT sector or the SUSY breaking sector, and thereby include low energy physics. This may be done, as we have mentioned above, by including in F contributions from the extra sectors, for example let us consider $F = f+h$ where f is the piece responsible for inflation defined above, $f = \mu^2[1-(\phi/v)^n]^m$ and $h = h(\psi^i)$ is the superpotential of the GUT sector. Using $G = g(\phi,\phi^*)+\psi^i\psi_i^*+\log|F|^2$, we find for the scalar potential V:

$$V = \exp(g+\psi^i\psi_i^*) \; [|f_\phi+g_\phi F|^2/g_\phi^\phi+|h_i+\psi_i^*F|^2-3|F|^2] \; . \qquad (3.24)$$

Since $h \ll \mu^2$, $F \approx f$ at large energy scales and we recover the expression for V used in our previous analyses. After the inflationary phase transition however, when $\phi = v$, f and f_ϕ vanishes and V is expressed entirely in terms of the GUT sector:

$$V \propto \exp(\psi^i\psi_i^*)[|h_i+\psi_i^*h|^2-3|h|^2] \; . \qquad (3.25)$$

In this way one can also wish to add a Polonyi term [22] to break the supersymmetry, $F_s = m_o^2(\beta+z)$ where $m_o \sim 10^{-8}$ is the scale of supersymmetry breaking and z is the Polonyi field responsible for the breaking.

IV. CONCLUSIONS

The necessity of separating the inflation sector from the GUT sector is unattractive in the sense that it requires new physics to describe an inflationary phase transition in the early Universe. Despite our complete uncertainty regarding this new sector, it is of interest to determine whether or not an inflationary model can be derived using only a single new field and no relative fine tunings among the couplings. In a minimal N=1 supergravity theory, it is clear that this is not possible. Namely, one cannot write down a superpotential of the form

$$f(\phi) = \mu^2 g(\phi) \tag{4.1}$$

where the couplings in $g(\phi)$ are all $O(1)$ and μ^2 is determined from the constraint coming from density perturbations. Such a model fails in that the high temperature correction to the scalar potential derived from eq. (4.1) has no minimum near $\phi = 0$. Hence there is no reason to expect that the initial conditions would single out $\phi = 0$ rather than any other point.

The use of finite temperature corrections can remove the arbitrariness of $\phi_{initial}$ by taking $\phi_{initial} = \min(V_T)$. Doing this however requires an additional assumption. It requires that at some early epoch thermal equilibrium was once established. Below M_p, this seems unlikely as self couplings are too small to compete with the expansion rate of the Universe. One must assume therefore, that at scales above M_p, new physics was responsible for the thermal distribution. Inflation will then follow if the thermal effects pick out $\phi = 0$ as the initial condition. Whether or not $\phi=0$ is chosen has been the subject of recent debate [23]. It has been argued however that in models in which $\langle\phi\rangle = v \sim M_p$, i.e. primordial inflation, proper initial conditions are obtained [24].

One is left therefore, with three alternatives: 1) ignore the thermal constraint; 2) find a scenario which does not require it; or 3) try to find a model which satisfies it. The first choice requires choosing by hand initial conditions and one can write a successful model which is very simple and satisfies all the zero temperature constraints, as was discussed in Section II. To alleviate the arbitrariness in the inflationary scenario, we prefer the second two alternatives.

There has been one attempt to find a scenario which bypasses the thermal constraint. This is known as chaotic inflation [25]. In chaotic inflation, one utilizes the fact that at very high "temperatures", there is a good probability that ϕ will be very far from $\phi = 0$. Indeed $\phi \gtrsim M_p$ is possible so long as $V(\phi) < M_p^4$. As ϕ begins to settle to its local minimum (given for example by a $\lambda\phi^4$ potential) inflation may occur. If we take the example of a $\lambda\phi^4$ potential, we need $\lambda \sim 10^{-12}$ in order not to overproduce density perturbations [26,27]. Inflation occurs however only when the Lagrangian is dominated by the potential energy rather than the kinetic terms. Therefore

$$\dot{\phi}^2 + (\partial_\mu \phi)^2 \lesssim V \lesssim M_p^4 \tag{4.2}$$

If we assume for some reason that ϕ begins at rest (already a bad assumption at the Planck scale) $\dot{\phi} = 0$. If we then take ϕ to be smooth over a horizon scale $\Delta L \sim H^{-1} \sim M_p^{-1}$ then

$$\Delta\phi < \frac{\lambda^{1/2}\phi^2}{M_p} < M_p \tag{4.3}$$

which requires $\phi_{initial}$ as large as $10^3\ M_p$ with $\Delta\phi < M_p$. In short one would expect this model to be dominated by kinetic energy terms rather than the potential. If one assumes however that the field was smooth over a scale $\Delta L \sim 10^3 H^{-1} \sim 10^3 M_p^{-1}$ one has $\Delta\phi \lesssim 10^3 \lambda^{1/2}\phi^2/M_p \lesssim 10^3 M_p$ or $\phi_{initial} \sim \Delta\phi \sim 10^3 M_p$ [27]. Smoothness over many horizon scales ($10^3 H^{-1}$) seems equivalent in choosing by hand the initial conditions of our first choice.

In this paper we have taken the route of our third choice, i.e. to satisfy the thermal constraint. In the context of minimal supergravity, one can either add additional fields and/or make severe fine tunings in the couplings to satisfy all constraints. This has apparently been shown to be possible. We preferred however to rest with a single field and no relative fine tunings by exploring the effects of a theory with non-minimal kinetic terms. We find that indeed this approach is possible.

Finally we note that there have been other attempts at inflation utilizing non-minimal supergravity models based on a SU(N,1) symmetry [28]. In these models, the Kähler potential has the basic form [29]

$$G = -3\ \ln(f(z,z^*) - \phi^i\phi_i^*/3) + \ln|F|^2 \tag{4.4}$$

where z is the Polonyi field responsible for SUSY breaking and ϕ^i (1...N) contain the inflaton and matter fields. These theories necessarily involve the mixing of the z field with the matter fields and cannot be discussed in terms of single field inflation. Care again must be taken to solve the thermal constraint. In the so-called maximally symmetric models [30], although the scalar potential is symmetric about $\phi = 0$ the thermal potential contains a linear term so that $\phi_{initial} \neq 0$. Simpler versions of this model however can be made to satisfy the thermal constraint with the choice [28]

$$F(\phi) = \mu^2(\phi - \phi^4/4) \tag{4.5}$$

In conclusion, we have shown that there exists a class of single field inflationary models which satisfy both the zero and finite temperature constraints. By utilizing non-minimal kinetic energy terms one can obtain the additional freedom to satisfy these constraints. In these models, the standard low energy N=1 supergravity theory is left unaltered.

Appendix

In this appendix we demonstrate that the field transformation carrying the "non physical inflaton" ϕ into a "physical inflaton" Φ does not alter our results.

The Kähler potential G of eq (3.21) implies a kinetic term in the Lagrangian

$$L_{KT} = G^{\phi}_{\phi}\partial_{\mu}\phi\partial^{\mu}\phi/2 \qquad \text{A.1}$$

where

$$G^{\phi}_{\phi} = 1+(1-\frac{ac}{3})\phi\phi^{*}+\frac{b}{2}(\phi^{2}+\phi^{*2})+\frac{1}{2}(c+\frac{N}{a})\ (\phi^{2}\phi^{*}+\phi\phi^{*2}) \quad . \qquad \text{A.2}$$

If $\phi = \phi_R + i\phi_I$, then

$$\begin{aligned} G^{\phi}_{\phi}d\phi d\phi^{*} &= G^{\phi}_{\phi}(d\phi_R^2+d\phi_I^2) \\ &= (1+\alpha\phi_R^2+\beta\phi_I^2+\gamma\phi_R^3+\gamma\phi_R\phi_I^2)(d\phi_R^2+d\phi_I^2) \qquad \text{A.3} \\ &= d\phi_R^2+d\phi_I^2+\text{ (terms of higher order)} \end{aligned}$$

where the terms of higher order have the form $\phi_R^2 d\phi_R^2$ etc. It follows therefore that to lowest order the new fields Φ_R, Φ_I are identical to ϕ_R and ϕ_I. Therefore we find for example for

$$\frac{\partial V}{\partial \phi_R} = \frac{\partial V}{\partial \phi_R}\frac{\partial \phi_R}{\partial \Phi_R} + \frac{\partial V}{\partial \phi_I}\frac{\partial \phi_I}{\partial \Phi_R} \qquad , \tag{A.4}$$

and evaluated at $\Phi = 0$ this vanishes because $\partial V/\partial \phi_R$ and $\partial V/\partial \phi_I$ vanishes. Also

$$\frac{\partial^2 V}{\partial \Phi_R^2} = \frac{\partial^2 V}{\partial \phi_R^2}\left(\frac{\partial \phi_R}{\partial \Phi_R}\right)^2 + 2\frac{\partial^2 V}{\partial \phi_R \partial \phi_I}\frac{\partial \phi_I}{\partial \Phi_R}\frac{\partial \phi_R}{\partial \Phi_R} + \frac{\partial V}{\partial \phi_R}\frac{\partial^2 \phi_R}{\partial \Phi_R^2} + \frac{\partial^2 V}{\partial \phi_I^2}\left(\frac{\partial \phi_I}{\partial \Phi_R}\right)^2 + \frac{\partial V}{\partial \phi_I}\frac{\partial^2 \phi_I}{\partial \Phi_R^2} \tag{A.5}$$

which also vanishes at the origin because

$$\frac{\partial^2 V}{\partial \phi_R^2}, \frac{\partial \phi_I}{\partial \Phi_R}, \frac{\partial V}{\partial \phi_R}, \frac{\partial^2 \phi_I}{\partial \Phi_R^2} \tag{A.6}$$

all vanish. Similarly one sees that $\partial^2 V/\partial \Phi_I^2 > 0$ because $\partial^2 V/\partial \phi_I^2 > 0$.

By carrying out the field transformation in the vicinity of the minimum v of V one similarly finds that all constraints at v are satisfied.

References

[1] A. Guth, Phys. Rev. D 23 (1981) 347.

[2] E. Kolb and S. Wolfram, Ap. J. 239 (1980) 428; K. Sato, Phys. Lett. 99B (1981) 66; D. Kazanas, Ap. J. 241 (1980) L59.

[3] A. D. Linde, Phys. Lett. 108B (1982) 389; A. Albrecht and P. Steinhardt, Phys. Rev. Lett. 48 (1982) 1220.

[4] S. W. Hawking, Phys. Lett. 115B (1982) 295; A. H. Guth and S.-Y. Pi, Phys. Rev. Lett. 49 (1982) 1110; A. A. Starobinski, Phys. Lett. 117B (1982) 175; J. M. Bardeen, P. J. Steinhardt, and M. S. Turner, Phys. Rev. D 28 (1983) 679.

[5] A. D. Linde, Phys. Lett. 116B (1982) 335; J. Breit, S. Gupta and A. Zaks, Phys. Rev. Lett. 51 (1983) 1007.

[6] J. Ellis, D. V. Nanopoulos, K. Olive and K. Tamvakis, Phys. Lett. 118B (1982) 335.

[7] J. Ellis, D. V. Nanopoulos, K. Olive and K. Tamvakis, Nucl. Phys. B221 (1983) 224.

[8] A. Albrecht, S. Dimopoulos, W. Fischler, E. Kolb, S. Raby, and P. J. Steinhardt, Nucl. Phys. B229 (1983) 528.

[9] P. J. Steinhardt and M. S. Turner, Phys. Rev. D 29 (1984) 2105.

[10] E. Cremmer, B. Julia, J. Scherck, S. Ferrara, L. Girardello and P. Van Nieuwenhuizen, Phys. Lett. 79B (1978) 231 and Nucl. Phys. B147 (1979) 105; E. Cremmer, S. Ferraro, L. Girardello and A. Van Proeyen, Phys. Lett. 116B (1982) 231 and Nucl. Phys. B212 (1983) 413.

[11] D. V. Nanopoulos, K. A. Olive, M. Srednicki, and K. Tamvakis, Phys. Lett. 123B (1983) 41.

[12] R. Holman, P. Ramond and G. G. Ross, Phys. Lett. 137B (1984) 343.

[13] G. D. Coughlan, W. Fischler, E. W. Kolb, S. Raby, and G. G. Ross, Phys. Lett. 131B (1983) 59.

[14] G. Gelmini, D. V. Nanopoulos, and K. A. Olive, Phys. Lett. 131B (1983) 53.

[15] B. Ovrut and P. J. Steinhardt, Phys. Lett. 133B (1983) 161.

[16] K. A. Olive and M. Srednicki, Phys. Lett. 148B (1984) 437; P. Binetruy and M. K. Gaillard, Nucl. Phys. B (in press) 1985 and Berkeley preprint UCB-PTH-85/2 (1985).

[17] B. Ovrut and P. J. Steinhardt, Phys. Rev. Lett. 53 (1984) 732; P. Binetruy and S. Mahajan, LBL preprint LBL-18566 (1984).

[18] e.g. M. Claudson, L. Hall, and I. Hinchliffe, Phys. Lett. 130B (1983) 260; E. Cremmer, S. Ferrara, C. Kounnas, and D. V. Nanopoulos, Phys. Lett. 133B (1983) 287.

[19] J. Ellis, D. V. Nanopoulos, K. A. Olive, and K. Tamvakis, Phys. Lett. 120B (1983) 331.

[20] L. F. Abbot, E. Fahri, and M. Wise, Phys. Lett. 117B (1982) 29; D. V. Nanopoulos, K. A. Olive, and M. Srednicki, Phys. Lett. 127B (1983) 30.

[21] J. Ellis, K. Enqvist, G. Gelmini, C. Kounnas, A. Masiero, D. V. Nanopoulos, A. Yu. Smirnov, Phys. Lett. 147B (1984) 27.

[22] J. Polonyi, Budapest preprint 1977-93 (1977).

[23] G. F. Mazenko, W. G. Unruh and R. M. Wald, Phys. Rev. D 31 (1985) 273.

[24] A. Albrecht and R. Brandenberger, preprint NSF-ITP-84-146 (1984); G. D. Coughlan and G. G. Ross, Oxford preprint (1984); L. Jensen and K. A. Olive, in preparation 1985.

[25] A. D. Linde, Phys. Lett. 129B (1983) 177.

[26] A. S. Goncharov and A. D. Linde, Class. Quantum Grav. 1 (1984) L75.

[27] H. Kodama, Tokyo preprint UTAP-9, 1984.

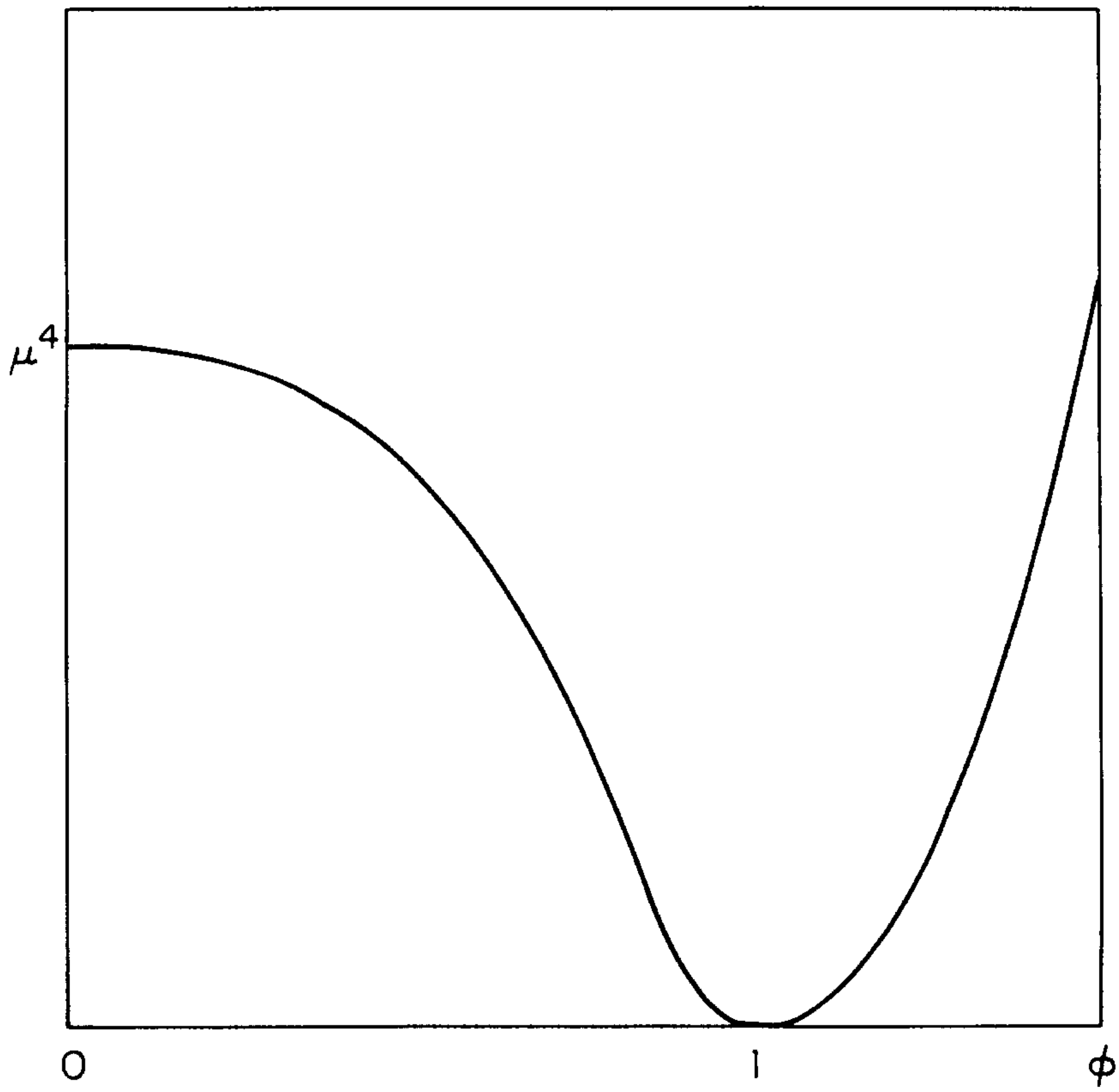
[28] J. Ellis, K. Enqvist, D. V. Nanopoulos, K. A. Olive, and M. Srednicki, Phys. Lett. 152B (1985) 175.

[29] J. Ellis, C. Kounnas, and D. V. Nanopoulos, Nucl. Phys. B241 (1984) 406 and B247 (1984) 373.

[30] G. Gelmini, C. Kounnas and D. V. Nanopoulos, CERN preprint Th. 3777 (1983); K. Enqvist and D. V. Nanopoulos, Phys. Lett. 142B (1984) 349 and CERN preprint Th. 3935 (1984); C. Kounnas and M. Quiros, Phys. Lett. 151B (1985) 189; K. Enqvist, D. V. Nanopoulos, M. Quiros, and C. Kounnas, CERN preprints Th. 4027 and Th. 4048.

Figure Caption

Figure 1: The T=0 scalar potential in the $\phi=\phi^*$ (ϕ_R) direction for the parameters given by eq. (3.15).



μ^4
V
0
1
ϕ